

Feedback and rate asymmetry of the Josephson junction noise detector

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The Josephson junction noise detector measures the skewness of non-Gaussian noise via the asymmetry of the rate of escape from the zero-voltage state upon reversal of the bias current. The feedback of this detector on the noise generating device is investigated in detail. Concise predictions are made for a second Josephson junction as noise generating device. The strong nonlinearity of this component implies particularly strong feedback effects, including a change in sign of the rate asymmetry as the applied voltage approaches twice the superconducting gap.

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In the last decade there have been extensive theoretical studies of non-Gaussian noise generated by nonlinear electronic nanostructures.^{1,2} This new field of full counting statistics (FCS) has put forward numerous predictions indicating that the higher order noise cumulants contain valuable information about the electronic transport mechanisms within the noise generating element that are not accessible from more standard measurements of the conductance and the noise power alone. A few years ago, first experimental observations of non-Gaussian current noise have been reported,^{3,4} and more recently quantum point contacts^{5,6} and Josephson junctions⁷⁻⁹ have emerged as promising on-chip noise detectors.

The current experimental efforts mainly focus on reliable measurements of the third noise cumulant C_3 , the skewness of the noise. A Josephson junction (JJ) in the zero-voltage state can detect the skewness of a noise current passing the detector since C_3 is odd under time reversal and thus leads to an asymmetry of the rate of escape from the zero-voltage state of the JJ when the direction of the bias current is reversed.⁷⁻⁹ After some primary suggestions for JJ noise detectors,¹⁰⁻¹⁴ concrete theoretical studies for the setup used in experiments have started recently.¹⁵⁻¹⁷

Now, that the theoretical tools for predictions for JJ noise detectors with realistic parameters are available, we shall address here the feedback of the noise detector on the noise generating device and show that this feedback can modify the rate asymmetry, the quantity determined experimentally, quite considerably. A detailed understanding of this feedback is essential for reliable data analysis. Already the first successful measurement of C_3 by Reulet *et al.*³ had to be corrected by large environmental effects associated with the leads connecting the noise generating element at cryogenic temperatures and the electronics at room temperature.^{18,19} However, feedback effects are also of relevance for on-chip detectors where the feedback is of a different character: the very concept implies two mesoscopic devices, detector and measured device, which interact on the same chip and thus have to be treated on the same footing. To be concrete, we shall specifically investigate the detection of non-Gaussian noise generated by another JJ biased by a voltage V_N with eV_N close to twice the gap Δ of the superconductor. Because of the strong nonlinearity of the noise generating device in this region, feedback of the detector leads to particularly pronounced modifications of the rate asymmetry.

A typical realization of a JJ noise detector is sketched in Fig. 1: a JJ with capacitance C and critical current I_c is connected by two branches which act as noise sources. A bias voltage V_B is applied to the first branch with an Ohmic resistor R_B in series with the junction. This branch allows us to control the detector by fixing the average bias current, applying current pulses, and reading out voltage signals. A second voltage V_N is applied to another branch which includes a nonlinear noise generating conductor, again in series with the detector. This noise generating element can most generally be characterized by its charge-transfer statistics, its voltage dependent current cumulants $\{C_1(V), C_2(V), \dots\}$. Current experimental setups are more sophisticated, but the circuit diagram in Fig. 1 captures the essentials of a JJ on-chip noise detector.

The state variables of the JJ detector are the charge Q on the junction capacitance C and the phase difference φ between the order parameters of the superconductors on either side of the tunnel barrier. The phase dynamics of the detector can be described by classical physics since the effective temperature determined by the strength of Gaussian noise from the two branches (see below) is typically much larger than the crossover temperature below which macroscopic quantum tunneling of the phase²⁰ becomes relevant. The stochastic dynamics of the setup can then be described in terms of a Hamiltonian \mathcal{H} (Ref. 21) which depends on the state variables Q, φ and the conjugate thermodynamic forces λ, μ . The Hamiltonian of the stochastic theory has to be distinguished from the microscopic Hamiltonian. \mathcal{H} depends only on gross variables of the system and is measured in the same

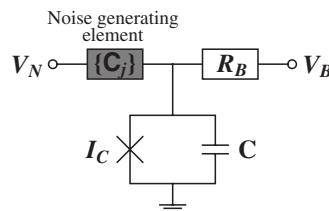


FIG. 1. Circuit diagram of a JJ noise detector: a JJ with critical current I_c and capacitance C is biased in a twofold way. The branch to the right sets an Ohmic resistor R_B in series with the junction and is biased by the voltage V_B . The branch to the left is biased by a voltage V_N and includes the noise generating nonlinear element characterized by its cumulants C_j .

units as the rate of change in entropy [$k_B\omega$] rather than energy [$\hbar\omega$]. For the setup in Fig. 1 $\mathcal{H}=\mathcal{H}_D+\mathcal{H}_B$, where

$$\mathcal{H}_D = \frac{2e}{\hbar} \frac{Q}{C} \mu - I_c \sin(\varphi) \lambda \quad (1)$$

describes plasma oscillations of the JJ detector, and

$$\mathcal{H}_B = \left[\frac{1}{R_B} \left(V_B - \frac{Q}{C} \right) + C_1 \right] \lambda + \left[\frac{T}{2R_B} + \frac{C_2}{4k_B} \right] \lambda^2 + 2k_B \sum_{j=3}^{\infty} \frac{C_j}{j!} \left(\frac{\lambda}{2k_B} \right)^j \quad (2)$$

adds the two biasing branches. While the branch with Ohmic resistor R_B generates Johnson-Nyquist noise and has only two nonvanishing cumulants $C_1^B = (V_B - Q/C)/R_B$ and $C_2^B = 2k_B T/R_B$, the other branch is characterized by the cumulants C_j that are taken at voltage $V_N - Q/C$.

The canonical equations of motion following from the Hamiltonian \mathcal{H} determine the most probable path between a given initial state i and a final state f , and the transition probability between these states may be written as a path integral,²¹

$$p_{i \rightarrow f} = \int D[Q, \varphi, \lambda, \mu] \exp \left\{ - \frac{1}{2k_B} A[Q, \varphi, \lambda, \mu] \right\}, \quad (3)$$

where the action functional is given by

$$A[Q, \varphi, \lambda, \mu] = \int_0^t ds [\dot{Q} \lambda + \dot{\varphi} \mu - \mathcal{H}(Q, \varphi, \lambda, \mu)]. \quad (4)$$

During escape of the detector from the zero-voltage state, the dimensionless quantity $e\lambda/k_B$ can be shown¹⁷ to be always much smaller than 1. Furthermore, the voltage across the JJ detector,

$$V_J = \frac{Q}{C} = \frac{\hbar}{2e} \dot{\varphi}, \quad (5)$$

that builds up under noise activation is proportional to λ . Keeping in Eq. (2) only terms that are at most of third order in λ and Q/C , we obtain $\mathcal{H}=\mathcal{H}_2+\mathcal{H}_3$, where

$$\mathcal{H}_2 = \frac{2e}{\hbar} \frac{Q}{C} \mu - I_c \sin(\varphi) \lambda + \left(I_t - \frac{1}{R_{\parallel}} \frac{Q}{C} \right) \lambda + \frac{T_{\text{eff}}}{2R_{\parallel}} \lambda^2 \quad (6)$$

describes the stochastic dynamics in presence of Gaussian noise, while

$$\mathcal{H}_3 = \frac{1}{24k_B^2} C_3^N \lambda^3 - \frac{1}{4k_B} \frac{\partial C_2^N}{\partial V_N} \frac{Q}{C} \lambda^2 + \frac{1}{2} \frac{\partial^2 C_1^N}{\partial V_N^2} \frac{Q^2}{C^2} \lambda \quad (7)$$

includes the leading-order effects of non-Gaussian noise. Here we have defined the total bias current through the detector

$$I_t = \frac{V_B}{R_B} + C_1^N \quad (8)$$

and the parallel (differential) resistance

$$\frac{1}{R_{\parallel}} = \frac{1}{R_B} + \frac{\partial C_1^N}{\partial V} \quad (9)$$

of the circuit. Moreover, we have introduced the effective temperature

$$T_{\text{eff}} = R_{\parallel} \left[\frac{T}{R_B} + \frac{C_2^N}{2k_B} \right] \quad (10)$$

characterizing the strength of Gaussian noise. The C_j^N are the cumulants C_j taken at voltage V_N .

In terms of the effective parameters introduced above, the leading-order Hamiltonian \mathcal{H}_2 has the standard form for a biased JJ described by the resistively and capacitively shunted junction model.²² The phase φ moves in the ‘‘tilted washboard’’ potential,

$$U(\varphi) = - \frac{\hbar I_c}{2e} [\cos(\varphi) + s\varphi], \quad (11)$$

driven by Gaussian noise. Here $s=I_t/I_c$ is the dimensionless bias current. For $0 < s < 1$, the potential has extrema in the phase interval $[0, 2\pi]$ at $\varphi_{\text{well, top}} = \arcsin(s) = \frac{\pi}{2} + \delta$ with $\delta \approx \sqrt{2(1-s)}$ for $1-s \ll 1$. When the JJ is trapped in the state $\varphi_{\text{well}} = \frac{\pi}{2} - \delta$, the average voltage V_J across the junction vanishes. This zero-voltage state is metastable, and to escape from the well, the junction needs to be activated to the barrier top at $\varphi_{\text{top}} = \frac{\pi}{2} + \delta$ by noise forces. The weak non-Gaussian noise described by \mathcal{H}_3 also gives a contribution to this process, and the rate of escape Γ may be written as

$$\Gamma = f e^{-(B_2+B_3)}, \quad (12)$$

where f is the prefactor of the rate, and the exponential factor is determined by the action of the most probable escape path, which is a solution of the canonical equations of motion following from the Hamiltonian with the boundary conditions $\varphi(t=-\infty) = \varphi_{\text{well}}$ and $\varphi(t=+\infty) = \varphi_{\text{top}}$. The exponential rate factor has a dominant contribution,

$$B_2 = \frac{\Delta U}{k_B T_{\text{eff}}}, \quad (13)$$

arising from \mathcal{H}_2 , where $\Delta U = U(\varphi_{\text{top}}) - U(\varphi_{\text{well}})$ is the barrier height of the metastable well. The correction B_3 due to non-Gaussian noise may be evaluated by treating \mathcal{H}_3 as a perturbation. Following the lines of reasoning in Ref. 17 one finds

$$B_3 = - \frac{1}{(k_B T_{\text{eff}})^3} \left(\frac{\hbar}{2e} \right)^3 C_{3, \text{eff}} J, \quad (14)$$

where

$$C_{3, \text{eff}} = C_3^N - 3k_B T_{\text{eff}} \frac{\partial C_2^N}{\partial V_N} + 3(k_B T_{\text{eff}})^2 \frac{\partial^2 C_1^N}{\partial V_N^2} \quad (15)$$

is the *effective* third noise cumulant. The second and third terms in Eq. (15) arise from the feedback of the JJ on the noise generating device, which is a consequence of the finite voltage V_J arising during escape. The quantity

TABLE I. Circuit parameters used for the calculations of the quantities displayed in Figs. 2 and 3.

$\Delta=200 \mu\text{eV}$	Superconducting gap
$C_N=1 \text{ fF}$	Capacitance of noise generating JJ
$R_t=30 \text{ k}\Omega$	Tunneling resistance of noise generating JJ
$R_N=200 \Omega$	Lead resistance of non-Gaussian noise branch
$R_B=200 \Omega$	Lead resistance of biasing control branch

$$J = -\frac{1}{6} \int_{-\infty}^{\infty} dt \dot{\varphi}_{\text{relax}}^3(t) \quad (16)$$

is expressed in terms of the deterministic trajectory $\varphi_{\text{relax}}(t)$ starting at $t=-\infty$ at the barrier top φ_{top} and relaxing (in the absence of noise forces) to the well bottom φ_{well} reached at time $t=\infty$.

The coefficient B_2 is even under time reversal, while B_3 is odd, like the third noise cumulant. Accordingly, when the direction of the bias current is reversed, the rate coefficient shows an asymmetry which is a signature of non-Gaussian noise. The deviation of the rate ratio,

$$\frac{\Gamma(I_t)}{\Gamma(-I_t)} = \exp[-2B_3(I_t)] \approx 1 - 2B_3(I_t), \quad (17)$$

from 1 allows for an experimental determination of B_3 despite the fact that the corrections due to non-Gaussian noise are typically small.

In the following we will focus on a specific noise generating element, namely, a JJ with normal-state tunneling resistance R_t and capacitance C_N biased by a voltage V_N which is tuned close to twice the gap Δ of the superconductor. In view of the lead to the voltage source, this noise generating element is in series with an approximately Ohmic lead resistance R_N in the range of a few 100 Ω . For a setup with parameters around those indicated in Table I, the noise generating JJ has a very low subgap current, and for $eV > 2\Delta$ the current is carried by tunneling quasiparticles. We can then employ the semiconductor model,²³ where the superconductor is described in terms of quasiparticles with a density of states (DOS) $N_S(E)$ given by the familiar BCS expression,

$$\frac{N_S(E)}{N_0} = \frac{|E|}{\sqrt{E^2 - \Delta^2}}, \quad (18)$$

where N_0 is the normal-state DOS at the Fermi level. The mean current through the JJ can be written as

$$\langle I \rangle = C_1^{\text{qp}} = e(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}), \quad (19)$$

where the forward and backward quasiparticle tunneling rates across the junction interface are given by^{24,25}

$$\begin{aligned} \Gamma_{\rightarrow}(V) &= \frac{1}{e^2 R_t} \int dE dE' \frac{N_S(E) N_S(E' + eV)}{N_0^2} \\ &\times f(E)[1 - f(E' + eV)] P(E - E') \end{aligned} \quad (20)$$

and $\Gamma_{\leftarrow}(V) = \Gamma_{\rightarrow}(-V)$, respectively. Here, $f(E)$ is the Fermi distribution. The influence of the electromagnetic environ-

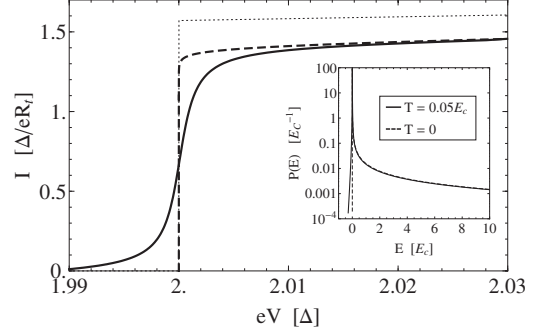


FIG. 2. I - V characteristic of the noise generating JJ at $T=50 \text{ mK}$ (solid line) and $T=0 \text{ K}$ (dashed line) in presence of DCB. The dotted line shows the steplike discontinuity for a JJ without DCB. The inset displays the $P(E)$ function.

ment is described in terms of the probability $P(E)$ that a tunneling quasiparticle loses the energy E to the environment. This dynamical Coulomb blockade (DCB) effect leads to a smearing of the expected discontinuity of $I(V)$ at $eV = 2\Delta$. Since the phase dynamics of the JJ detector is very slow on the time scale of quasiparticle tunneling in the noise generating JJ, the impedance of the electromagnetic environment is essentially given by the resistance R_N of the lead to the voltage source V_N . For this case of an Ohmic environment one has²⁵

$$\begin{aligned} P(E) &= \int_{-\infty}^{\infty} \frac{dt}{2\pi\hbar} e^{iEt/\hbar} \\ &\times \exp \left[2 \int_{-\infty}^{\infty} \frac{d\omega \text{Re} Z_{\text{eff}}(\omega)}{\omega R_K} \frac{e^{-i\omega t} - 1}{1 - e^{-\hbar\omega/k_B T}} \right], \end{aligned} \quad (21)$$

where $Z_{\text{eff}}(\omega) = R_N / (1 + i\omega R_N C_N)$ is the effective impedance and $R_K = \hbar/e^2$. For the set of parameters given in Table I, the resulting I - V characteristic of the noise generating JJ is shown in Fig. 2. The inset shows the function $P(E)$.

The cumulants of the quasiparticle tunneling current are readily determined from the tunneling rates [Eq. (20)]. One has²⁶

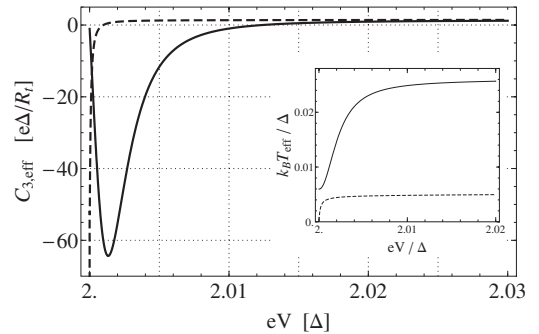


FIG. 3. The effective third cumulant for $T=50 \text{ mK}$ (solid line) and $T=0 \text{ K}$ (dashed line). The inset shows the effective temperature.

$$C_j^{\text{qp}} = e^j[\Gamma_{\rightarrow} + (-1)^j\Gamma_{\leftarrow}], \quad (22)$$

which implies, in particular, $C_3^{\text{qp}} = e^2 C_1^{\text{qp}}$. Although R_N has a significant effect on the noise cumulants via DCB, in the region $eV_N > 2\Delta$, its influence on the voltage across the noise generating JJ can safely be neglected since $R_N/R_I \ll 1$. Hence, the modifications of this voltage are essentially due to the detector feedback of order V_j/V_N discussed above. Accordingly, in Eqs. (10) and (15), we can set $C_j^N \approx C_j^{\text{qp}}(V_N)$.

The highly nonlinear behavior of the cumulants for $eV \sim 2\Delta$ strongly affects the effective temperature T_{eff} and the effective third cumulant $C_{3,\text{eff}}$. Both quantities are shown in Fig. 3 as functions of the applied voltage, again for the representative set of parameters given in Table I. While the skewness of the noise C_3 is always positive for positive I_r , the rate asymmetry (17), which is proportional to $C_{3,\text{eff}}$, passes through zero as V_N is decreased and it takes large negative values for voltages slightly above $2\Delta/e$. This change in sign

constitutes a pronounced feedback effect that should be easily observable experimentally.

In summary, we have shown that feedback of the JJ noise detector on the noise generating device can be very pronounced if the device under investigation is highly nonlinear. In particular, another JJ as noise source should allow for a detailed experimental study of this effect, which leads to a strongly enhanced asymmetry of the switching rate of the detector JJ and even a change in sign of the apparent noise skewness $C_{3,\text{eff}}$ as the voltage applied to the noise generating junction approaches twice the superconducting gap. The size of the feedback corrections sensitively depends on DCB of quasiparticle tunneling.

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